Pergamon

Int. J. Heat Mass Transfer. Vol. 38, No. 5, pp. 899–907, 1995

0017–9310/95 $9.50+0.00

0017–9310(94)00197–9

Optimization of radiative–convective arrays of pin fins including mutual irradiation between fins

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(*Received* 14 *December* 1993 *and in final form* 21 *June* 1994)

Abstract—This study investigates the optimal pin fin array of variable cross section for a given fin material per unit base area. The effects of conduction, convection and radiation heat transfer is studied including the mutual irradiation between a fin and all other fins and the base. The finite difference method is used to solve the resulting nonlinear-integral–differential equations. The effect of parameters such as fin spacing, profile, and emissivity on the fin array effectiveness is studied and the results are presented in a graphical form convenient for parametric study and design analysis.

INTRODUCTION

The ability to dissipate heat effectively has been an engineering interest for a long time. The art of accomplishing a task with minimum cost has always been an engineering concern. In many engineering applications, convection and radiation from the surface of an object to the surrounding environment alone will not provide sufficient means to dissipate the required heat flux. This is especially true for a gaseous heat transfer medium. One method of dissipating the necessary heat is to incorporate fins on the surface.

In many engineering applications the important optimization problem is to find the minimum volume per unit area (VPUA, V'') of fins on a surface necessary to dissipate a heat flux. The individual fin geometry and the spacing between the fins must be considered jointly to optimize the fin array.

This study considers an infinite flat surface which is required to dissipate a given heat flux. Fins of circular cross-section are attached to the plate in a triangular manner. Given the fluid and material properties and parameters and the geometry of the fin array, the heat flux dissipated will be determined. The results will be used to determine the fin profile, fin length, and fin spacing that will produce the maximum heat flux dissipation for a given VPUA. It will also investigate the importance of mutual irradiation among the fins and the base.

The books written by Kern and Kraus [1, 2] about extended surfaces are representative of the abundant information available on this topic. While the study of radiating–convecting fins is not new, it is still an area open to new developments and techniques. Renewed interest is due to the augmentation of heat transfer in the electronic industry and heat transfer in space where optimization is an important concern.

Due to the renewed interest in this field, Garimella and Eibeck [3] presented a recent study to determine the convective heat transfer coefficients for an array of protruding elements in forced convection.

Many studies have been performed to determine the optimum fin shape for convective and or radiative transfer from a single fin. Sonn and Bar-Cohen [4] found the optimal diameter for a constant cross-sectional area cylindrical fin exposed to convection. Coble [5] found the optimum fin shape for a variable area fin exchanging heat with the surroundings by convection and radiation. Wilkins [6] found the optimal shape for fins rejecting heat by convection and radiation. Razani and Zohoor [7] considered a combined radiation and convection environment and found the optimal geometry for a fin whose profile was correlated to the temperature at each axial location within the fin. Hrymak *et al.* [8] employed a finite element technique to determine the optimum shape for a fin of polynomial profile subjected to convective and radiative heat loss. This technique allowed design constraints such as the maximum allowable length of the fins to bound the solution. But in all of these studies, multiple fin interactions were not considered.

The following set of references considered fin to fin interactions, but each fin could only transfer radiation to at most two other fins. Schnurr *et al.* [9] found the optimal radiative fins, both circular and straight, that were attached to a cylinder. Their study included fins of both triangular and rectangular profile, but they did not consider fin profile as a parameter in the optimization. Schnurr [10] similarly found the optimal

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NOMENCLATURE

eff	fin array effectiveness
F	configuration factor
h	convection coefficient
k	thermal conductivity
l	length
L	length of fin
Pr	Prandtl number
q	heat flux
Q	rate of heat transfer
r	fin radius
R	equivalent cylinder radius
S_D	fin spacing, adjacent rows
S_L	longitudinal pitch
S_T	transverse pitch
T	temperature
v	velocity
V	volume
VPUA	volume per unit area
V''	volume per unit area
x	distance from the base.

Greek symbols

α	angle of view
β	profile parameter
δ	fin spacing
ε	emissivity
ξ	distance from base
μ	viscosity
ρ	density (various types)
σ	Stefan–Boltzmann constant
τ	radial distance from fin.

Subscripts

a	atmosphere
b	base plate
c	surrounding cylinder
eq	equivalent
f	fin
i	incoming
max	maximum
o	outgoing
pl	base plate
rad	radiation
Red	reduced temperatures.

radiative fin with triangular profile for an array of longitudinal fins around a cylinder. Karlekar and Chao [11] performed a study similar to Schnurr [10] using fins with a rectangular profile. Cox [12] considered an array of cylinders and included radiative exchange between the cylinders in a sophisticated manner; however he assumed that the cylinders could be approximated as infinitely long. Aihara *et al.* [13] experimentally found the average heat transfer coefficient for an array of pin fins. In order to find the average convective heat transfer coefficient, though, they had to calculate the radiative transfer from the fins. The authors of the paper admit that they know of no way to perform this calculation. Therefore they assumed, for their experiment, that all the surfaces were black (actual emissivity was 0.9).

THEORY

Figure 1 shows the geometry used in convection and radiation calculations, cone-shaped pin fins of base radius r_b and length L in an array of equilateral triangles. The separation distance between the fins is δ. The radius of the fin varies as a function of the distance from the base plate. For computational reasons, the base plate is approximated as infinite. The fins can transfer heat by conduction, convection, and radiation including mutual irradiation between the fins and the base.

Computing the heat flux dissipated by this array is a difficult matter when radiative heat transfer is considered since each fin not only sees the base, the atmosphere, and the fins closest to it, but also many fins further away whose view may be partially blocked by fins in other rows.

Calculating the heat dissipated by a fin in a multiple fin array would be a difficult and time consuming process if all possible fin to fin interactions were considered separately. Therefore it was assumed that the array of fins surrounding any one fin can be approximated by a cylinder with a radius R, where R may be a function of r, L and δ [see Fig. 1(c)]. Since the radius of the fin is allowed to vary as a function of x (distance from the plate), R may also be a function of x. Since the correct relationship to determine R is difficult to obtain, R will be used as a parameter to study its effect on the optimization of an array of pin fins including mutual irradiation.

The following assumptions were made:

(1) All surfaces are diffuse, gray and opaque.

(2) The optical properties are not functions of temperature.

(3) The thermal properties of the fins, the base plate and the convecting fluid are not functions of temperature.

(4) The radiation incident on a surface is diffuse.

(5) The temperature of a cross-section in the fin is constant.

(6) All the fins surrounding any one fin may be approximated as a surrounding cylinder with a constant radius.

(7) The temperature distribution along the surrounding cylinder is the same as in the fin.

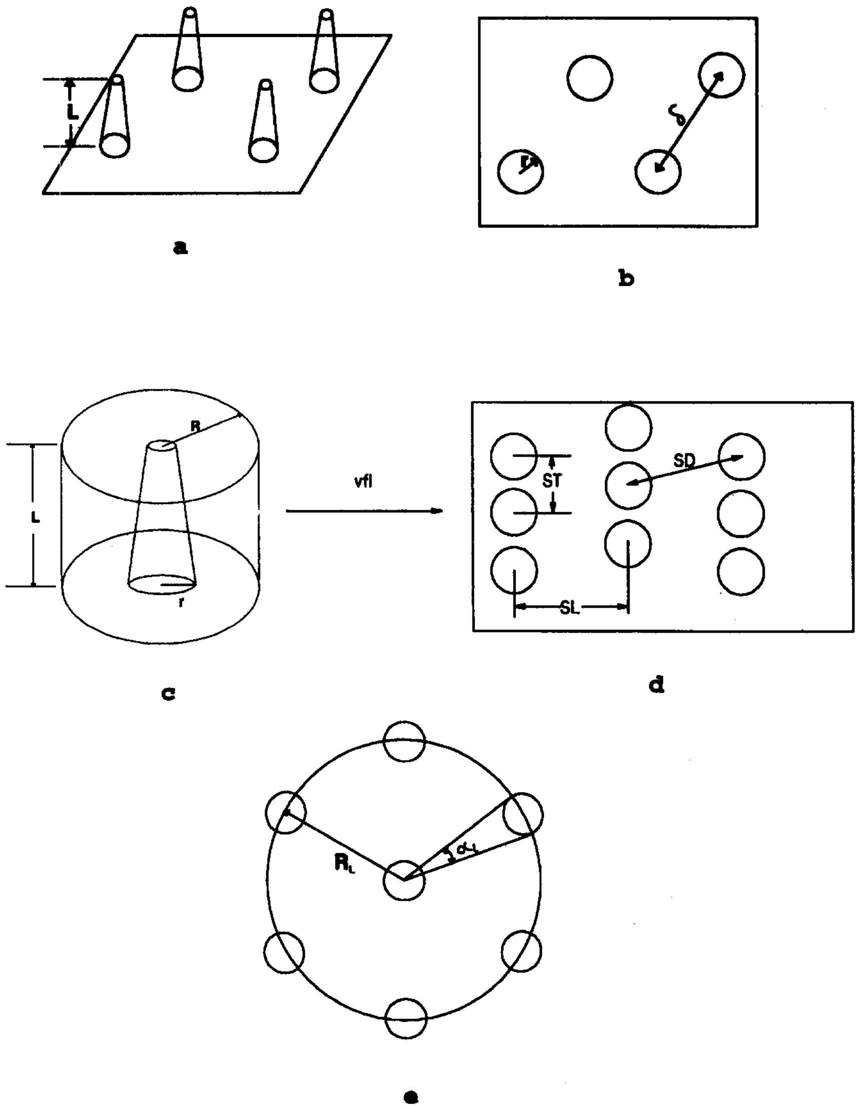

Fig. 1. Fin array geometry: (a) isometric view; (b) top view; (c) estimate of the fin array as an equivalent cylinder around each fin; (d) a general representation of a staggered fin array and (e) angle of view between fins.

(8) Convection and radiation heat transfer are decoupled.

(9) The temperature of the convecting fluid is constant. Although this assumption is not true when actual heat exchange is considered it would not change the objectives of this study.

(10) The heat flux dissipated by the area of the base plate associated with each fin due to radiation can be approximated by the heat flux dissipated by radiation from the area of the plate between the fin and the equivalent surrounding cylinder. The heat transferred from the base is then calculated by multiplying the heat flux by the actual area.

(11) Fin profile curvature is neglected with respect to radiative exchange calculations from a fin to itself. Each fin does not see itself in radiative heat exchange calculations.

(12) The base temperature remains isothermal.

The steady state energy balance for a convecting radiating fin can be written as

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[k\pi r^2(x)\frac{\mathrm{d}T_f(x)}{\mathrm{d}x}\right] - 2\pi r(x)h(x, L, r_b, \delta)[T_f(x) - T_a] - 2\pi r(x)[q_{f,rad,o}(x) - q_{f,rad,i}(x)] = 0. \quad (1)$$

In most heat transfer analysis concerning pin fins, the last term of equation (1) is neglected. Sometimes the effect of this term is taken into account, as a first approximation, by introducing an effective surrounding temperature for radiative heat transfer. The most important aspect of this investigation is to include the mutual radiative heat transfer between the fins. The radiation heat exchange consists of an enclosure made of four surfaces including the fin, the surrounding cylinder, the base, and the atmosphere. Formulation of radiative heat exchange in this enclosure can be written as

$$q_{\mathrm{f,rad,i}}(x) = \int_0^L q_{\mathrm{c,rad,o}}(\xi)\,\mathrm{d}F_{\mathrm{d}x-\mathrm{d}\xi} + \int_{r_b}^{R} q_{\mathrm{b,rad,o}}(\tau)\,\mathrm{d}F_{\mathrm{d}x-\mathrm{d}\tau} \quad (2)$$

$$q_{\mathrm{f,rad,o}}(x) = \varepsilon_f \sigma T^4_{\mathrm{Red,f}}(x) + (1-\varepsilon_f)\, q_{\mathrm{f,rad,i}}(x) \quad (3)$$

$$q_{\mathrm{c,rad,i}}(\xi) = \int_0^L q_{\mathrm{f,rad,o}}(x)\,\mathrm{d}F_{\mathrm{d}\xi-\mathrm{d}x} + \int_{r_b}^{R} q_{\mathrm{b,rad,o}}(\tau)\,\mathrm{d}F_{\mathrm{d}\xi-\mathrm{d}\tau} + \int_0^L q_{\mathrm{c,rad,o}}(\xi')\,\mathrm{d}F_{\mathrm{d}\xi-\mathrm{d}\xi'} \quad (4)$$

$$q_{\mathrm{c,rad,o}}(\xi) = \varepsilon_f \sigma T^4_{\mathrm{Red,f}}(\xi) + (1-\varepsilon_f)\, q_{\mathrm{c,rad,i}}(\xi) \quad (5)$$

$$q_{\mathrm{b,rad,i}}(\tau) = \int_0^L q_{\mathrm{f,rad,o}}(x)\,\mathrm{d}F_{\mathrm{d}\tau-\mathrm{d}x} + \int_0^L q_{\mathrm{c,rad,o}}(\xi)\,\mathrm{d}F_{\mathrm{d}\tau-\mathrm{d}\xi} \quad (6)$$

$$q_{\mathrm{b,rad,o}}(\tau) = \varepsilon_b \sigma T^4_{\mathrm{Red,b}} + (1-\varepsilon_b)\, q_{\mathrm{b,rad,i}}(\tau) \quad (7)$$

where $\mathrm{d}F_{\mathrm{d}x-\mathrm{d}\xi}$ is the configuration factor between a differential element on the fin, $\mathrm{d}x$, and a differential element on the surrounding cylinder, $\mathrm{d}\xi$. Also, remembering the seventh assumption,

$$T_{\mathrm{Red,c}}(\xi) = T_{\mathrm{Red,f}}(x). \quad (8)$$

Note that ξ and x both represent the distance from the base plate, with ξ referring to the surrounding cylinder and x referring to the fin, τ is distance from the center of the fin radially outward along the base plate, and the subscript "Red" refers to a reduced temperature ($T^4_{\mathrm{Red}} = T^4 - T^4_a$). Application of the reduced temperature has the effect of reducing the environmental temperature to absolute zero [14]. In calculating the radiative heat flow between the fin, the base plate, and the surrounding cylinder, the reduced temperature compensates for the radiation heat transfer through the atmosphere. Equations (1)–(8) provide a system of eight integral–differential equations which can be solved (numerically) for the eight unknowns $q_{\mathrm{f,rad,i}}$, $q_{\mathrm{f,rad,o}}$, $q_{\mathrm{c,rad,i}}$, $q_{\mathrm{c,rad,o}}$, $q_{\mathrm{b,rad,i}}$, $q_{\mathrm{b,rad,o}}$, $T_{\mathrm{Red,c}}(\xi)$ and $T_{\mathrm{Red,f}}(x)$. The total heat dissipated from the base plate per fin is the sum of the heat dissipated by the portion of the base plate associated with a fin by convection, conduction, and radiation:

$$Q = h(x, L, r_b, \delta)\left[\frac{1}{\rho_f} - \pi r_b^2\right][T_b - T_a] - k(T)\pi r_b^2 \frac{\mathrm{d}T_f(x=0)}{\mathrm{d}x} + \left[\frac{\dfrac{1}{\rho_f} - \pi r_b^2}{\pi(R^2 - r_b^2)}\right]\int_{r_b}^{R} [q_{\mathrm{b,r,o}} - q_{\mathrm{b,r,i}}]\,2\pi r\,\mathrm{d}r \quad (9)$$

where ρ_f is the number of fins per unit area on the plate. The heat flux from the plate, q, can then be calculated by multiplying the heat dissipated per fin with the fin density of the plate, ρ_f:

$$q = Q\rho_f. \quad (10)$$

The effectiveness of the fin array will be defined as the ratio of the heat flux dissipated by the base plate with the fin array to the base plate without the fin array:

$$\mathrm{eff} = \frac{q}{q_b} \quad (11)$$

For the triangular fin array shown in Fig. 1(a) and (b),

$$\rho_f = \left[\frac{\sqrt{3}}{2}\delta^2\right]^{-1}. \quad (12)$$

The radius of the fin will vary as a polynomial function of the distance from the base plate:

$$r(x) = r_b\left(1 - \frac{x}{L}\right)^{\beta} \quad (13)$$

where β is a free parameter which determines the fin profile and therefore its volume:

$$V_f = \int_0^L \pi r^2(x)\,\mathrm{d}x = \frac{\pi r_b^2 L}{2\beta + 1}. \quad (14)$$

The volume of fin material per unit area of base plate is

$$V'' = V_f \rho_f = \frac{2\pi r_b^2 L}{\sqrt{3}(2\beta+1)\delta^2} \quad (15)$$

or the radius at the base of the fin is

$$r_b = \delta\sqrt{\frac{\sqrt{3}V''(2\beta+1)}{2\pi L}}. \quad (16)$$

In order to solve these equations, it was necessary to approximate the convective heat transfer coefficient for the base of the plate and for the fin. For all other purposes, the length of the plate was considered infinite. But in order to calculate the convective heat transfer coefficient for the plate, a length was required. It was assumed that the convection coefficient for a flat plate is the same whether or not fins are attached. The convection coefficient for a flat plate will be found using the method presented in ref. [15].

Studies have been performed to determine the convective heat transfer coefficients for in-line and staggered arrays of protruding elements. Garimella and Eibeck [3] found the heat transfer coefficients for water cooling of the arrays and Sparrow and Ramsey [16] found the heat transfer coefficients for air cooling of staggered arrays. Both studies found that the flow was fully developed (heat transfer coefficients were independent of row number) after the fourth row. Sparrow and Ramsey [16] compared the results of their study with cross flow over tube banks. The taller the fins are, the closer they should approximate tube banks. They found that cross flow over tube banks was about 20% different from their test results for the shorter cylinders and the taller cylinders were within a few percent. In their study, they used values of fin height divided by fin diameter of 1, 2 and 3 for cylindrical pin fins. Sparrow *et al.* [17] continued the study to compare the advantages of using staggered and in-line fin arrays. They found that, for a fixed pumping power and surface area, the in-line array transferred more heat. For a fixed heat duty and flow rate, the staggered array minimized the fin surface area, but also increased the pumping power.

The tube bank correlations are well established in the literature. Therefore the convective heat transfer coefficient for the fin was found using the formulations for cross flow over tube banks [15] and [18]. These references used a general staggered fin arrangement. The spacing between fins in the same row is defined as the transverse pitch S_T, spacing between rows is defined as the longitudinal pitch S_L, and spacing between fins in adjacent rows is represented by S_D [see Fig. 1(d)]. For the arrangement of fins that is being used in this study, $S_T = S_D = \delta$ and $S_L = (\sqrt{3}/2)\delta$.

It should be pointed out that the convective heat transfer coefficient at each axial location is assumed to be the same as that for an array of cylinders with the radius equivalent to the radius of the fin at that location.

Perhaps the most challenging aspect of this study is the calculation of the radiative heat transfer between the fins. The radius of the surrounding cylinder is a very complex function of the fin arrangement.

Since the surrounding cylinder is used to estimate the radiative effect of the entire array of fins, the configuration factor from the fin to the surrounding cylinder should approximate the combined configuration factor from the fin to all other fins.

The configuration factor between one fin and all other fins was approximated by the following technique. For each row (row here refers to all fins that are equidistant from the fin of interest) of fins around a particular fin, the view factor, $F_{f\text{-}ci}$ between the fin and a surrounding cylinder of radius R_i that goes through the center of the fins in the ith row was first found [see Fig. 1(e)]. Next the angle of view that a fin in the ith row swept out was found using

$$\alpha_i = 2\tan^{-1}\left(\frac{r_{eq}}{R_i}\right) \tag{17}$$

where r_{eq} is the radius of a cylindrical fin that would produce the same volume as the actual fin. If n_i represents the number of fins that are located a distance R_i away, the configuration factor to the surrounding cylinder that approximates all other fins is represented by

$$F_{f-c} = \sum_{i=1}^{m} n_i \left(\frac{\alpha_i}{2\pi}\right) F_{f-ci} \tag{18}$$

where m is defined as the number of fin rows required to sweep out an entire circle:

$$\sum_{i=1}^{m} \frac{n_i \alpha_i}{2\pi} \approx 1. \tag{19}$$

For calculations in equations (18) and (19), rows which are completely blocked from the view of the fin of interest will not be used. The value of m that makes equation (19) just greater than one should be used. The fraction of the mth row that makes equation (19) equal one is then found. The last term in equation (18) is multiplied by this fraction.

A finite difference technique was employed to determine the heat flow through the fins. Each fin was divided into ten equal length sections. In order to determine the radiative heat flow the surrounding cylinder was also divided into similar equal length sections. To find the radiative heat transfer between each finite difference section, the configuration factors between the sections were found and the enclosure analysis method of Gebhart [14] was used.

RESULTS

In order to determine the effect of the important parameters on fin array optimization, the following values and ranges of values were used in the calculations:

Material	aluminum and stainless steel
Profile parameter	$\beta = 0$–20
Base temperature	$T_b = 350$–1000 K
Emissivity	$\varepsilon = 0, 1$; $\varepsilon_{Al} = 0.1$; $\varepsilon_{SS} = 0.161, 0.254$
Thermal conductivity	$k_{Al} = 117$; $k_{SS} = 16$; $k_{air} = 0.02624$ W m K^{-1}
Fin spacing	$\delta = 0.5, 2, 5$ cm
VPUA	$V'' = 10^{-4}$–10^{-2} m^3 m^{-2}
Convecting fluid	air
Fluid temperature	$T_a = 300$ K
Fluid viscosity	$\mu = 1.983 \times 10^{-5}$ kg m s^{-1}
Fluid Prandlt number	$Pr = 0.7080$
Fluid velocity	$v = 5, 10$ m s^{-1}
Plate length	$l_{pl} = 1$ m

Figure 2(a) shows the fin array effectiveness plotted against L/δ (the ratio of fin length to fin spacing) for $\beta = 0$ (constant cross-sectional area cylindrical fin) and $V''/\delta = 0.005$, 0.025, 0.05, 0.1, 0.15 and 0.2. According to this graph, as the length of the fin approaches the minimum length possible [length which causes $r_b \rightarrow \delta/2$ in equation (16)], the effectiveness of the array goes toward infinity. This is caused from the convection coefficient blowing up [15, 17]. The problem with using a radius this large compared to the fin spacing, however, is that the pump-

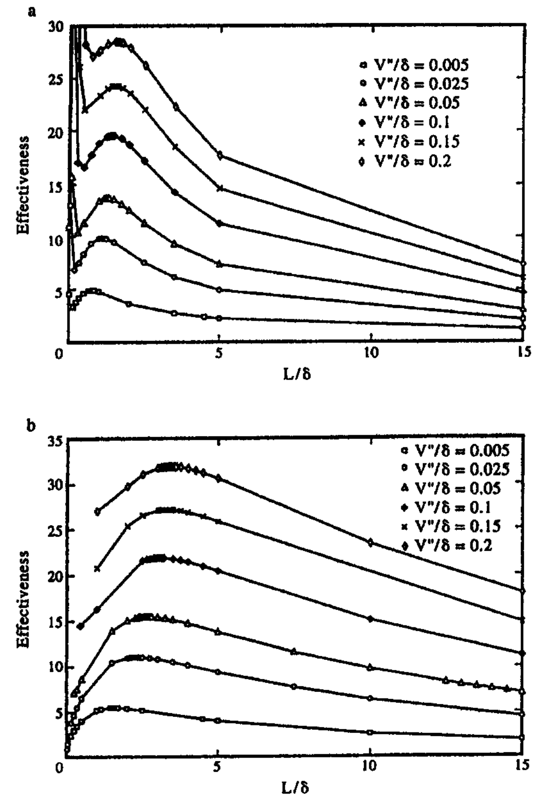

Fig. 2. Fin array effectiveness vs L/δ for air convecting over aluminum fins with $\varepsilon = 0.1$, $\delta = 2$ cm, $v = 5$ m s^{-1}, $T_a = 300$ K, $T_b = 500$ K. (a) $\beta = 0$ and (b) $\beta = 1.5$.

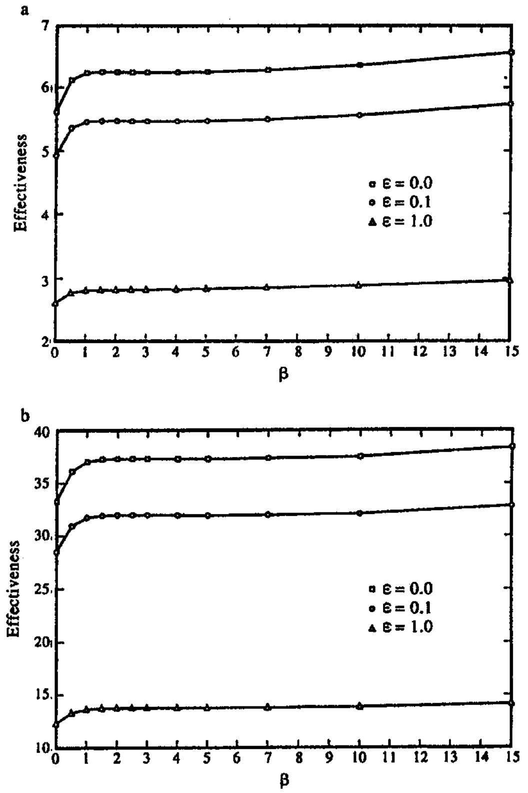

Fig. 3. Optimal fin array effectiveness versus β for air convecting over aluminum fins with $\delta = 2$ cm, $v = 5$ m s^{-1}, $T_a = 300$ K, $T_b = 500$ K. (a) $V'' = 1 \times 10^{-4}$ m^3 m^{-2} and (b) $V'' = 4 \times 10^{-3}$ m^3 m^{-2}.

ing power required to bring the convecting fluid to its desired velocity will also go to infinity. From Fig. 2(a), as the length of the fin is increased, the effectiveness initially decreases to a local minimum, then increases to a local maximum, and finally monotonically decreases eventually to a value of 1. It is interesting to note that there exists a local maximum for a given fin VPUA. The local maximum was used to determine the optimum length for the fins. The more material that was used, the more effective the fin array became. As more material was used, the optimal fin length and base radius both increased, but the base radius increased at a faster rate.

The problem was repeated for a fin profile parameter of 1.5. The results are plotted in Fig. 2(b). The reason the effectiveness did not get large for very small lengths of the fins, was due to the fact that the fins were only close at the base, and may not require a large pumping power either. Again, as the VPUA was increased, the fin base radius increased at a faster rate than the fin length. In this case it can also be demonstrated that a local maximum exists which helps the designer to find the optimum arrangement for the fin array. From Fig. 2(a) and (b) it can be seen that the maximum shifts to the larger values of L/δ as V''/δ increased. Comparisons also show that, for a given V''/δ, the maximum fin effectiveness increased with profile parameter $\beta = 1.5$ compared to $\beta = 0$.

The relationship between the fin profile parameter and the fin array effectiveness for emissivities of 0, 1 and 0.1 (actual emissivity) are shown in Fig. 3(a) for a fin VPUA of 1×10^{-4} m^3 m^{-2} and in Fig. 3(b) for a VPUA of 4×10^{-3} m^3 m^{-2}. In all cases, the emissivity of the base plate was set equal to the emissivity of the fins. For each profile parameter, the optimal fin length was used to calculate the effectiveness. For a set β, the optimal length for the fin did not change with the emissivity. The plots show that the effectiveness initially increased rapidly with increasing β, then decreased slightly, and then increased again. This study neglected the radiative exchange between a fin and itself and also made assumptions about the radiative view factors between the various elements based on an "averaged" fin radius for the part of the fin that was included in a particular configuration factor calculation. It also estimated the axial convection coefficient by assuming the radius of the entire fin was the radius at that axial location. Therefore the larger the curvature of the fins, the more error that was introduced into the results. For this reason, the optimal β that was used is located at the point on the graph where the effectiveness first comes to the local maximum. It should be pointed out that the nearly constant effectiveness for the larger values of β is very important in fin design analysis. This means that the

Table 1. Effect of radiation heat transfer between the fins and the base on heat flux and fin effectiveness (optimal eff for $V'' = 4\times10^{-3}$ m^3 m^{-2})

Row	ε_b	ε_f	β	L_{opt} [cm]	q_{opt} [W cm^{-2}]	eff
1	0	0	2	8	6.545	37.32
2	0.1	0.1	2	8	6.588	31.94
3	1	1	2	8	6.661	13.77
4	0	0	0	3.1	5.828	33.23
5	0.1	0.1	0	3.1	5.873	28.47
6	1	1	0	3.1	5.961	12.32
7	0	1	2	8	6.624	37.77
8	1	0	2	8	6.647	13.74
9	0	1	0	3	5.893	33.59
10	1	0	0	3	6.003	12.41

Table 2. Effect of radiation heat transfer between the fins and the base on heat flux and fin effectiveness (optimal eff for $V'' = 10^{-4}$ m^3 m^{-2})

Row	ε_b	ε_f	β	q_{opt} [W cm^{-2}]	eff
1	0	0	1.5	0.7330	3.891
2	0.161	0.161	1.5	0.7816	3.254
3	0.254	0.254	1.5	0.8078	2.995
4	1	1	1.5	1.0000	1.992
5	0	0	0	0.6824	3.891
6	0.161	0.161	0	0.7323	3.254
7	0.254	0.254	0	0.7599	2.995
8	1	1	0	0.9640	1.992

optimum fin is almost independent of the fin profile parameter for $\beta \geqslant 1$. This situation may change if one considers the pumping power or other design considerations such as the limitation on the length of the fin.

Figure 3(a) and (b) shows that as the emissivity of the fins and the base was increased, the effectiveness of the fin array decreased. Tabular data is contained in the first six rows of Table 1 for Fig. 3(b). Note that the same emissivity was used for the base as was used for the fins. The actual heat flux from the fin array increased if the emissivity was increased, but the effectiveness of the fin array went down. As the emissivity of the base goes up, the ability of the base to dissipate heat increases appreciably compared to convection heat transfer, especially if the heat transfer coefficient by convection is small. It is interesting to note that the effect of the emissivity is not linear between $\varepsilon = 0$ and $\varepsilon = 1$. The results of considering the effects of emissivity for $0 < \varepsilon < 1$ showed that the heat flux is closer to the black body heat flux than a linear relationship would produce. For $\beta = 2$, a linear relationship between $\varepsilon = 0$ and $\varepsilon = 1$ would predict an optimum heat flux of 6.557 for $\varepsilon = 0.1$ while the actual value was 6.588.

There are two effects caused by increasing the emissivity of the fins. The first effect causes the fin array effectiveness to decrease by transferring more heat from the base to the fins instead of to the atmosphere which is at a lower temperature. The second effect is to increase the fin array effectiveness by the fins being able to transfer more heat to the surroundings. In Table 1, comparing row 1 to 7, row 4 to 9, and row 3 to 8 shows that for these cases, as the base emissivity remains fixed, the effect of increasing the emissivity of the fins is to increase the effectiveness of the fin array. Comparison of row 10 to 6 shows that for this case the effect of increasing the emissivity of the fin with fixed base emissivity was to decrease the fin array effectiveness. Furthermore, the differences in the results caused by considering the emissivity of the fins for this problem were small. Comparison of rows 1 to 3 and 4 to 6 shows that neglecting radiation could decrease the heat flux from the fin array by about 2% in this case. The results for $V'' = 1\times10^{-4}$ m^3 m^{-2} are presented in Table 2. Comparisons of row 1 to 3 and row 4 to 6 show that, for this case, the effect of neglecting radiation could decrease the heat flux from the fin array by 19 and 22%, respectively. As the VPUA is decreased, the effects of emissivity become more pronounced. This is due to the lower portions of the fins being able to radiate more to the upper portions of the fins and to the atmosphere compared to the larger values of VPUA (since $\delta/2 - r$ will be larger for lower values of VPUA). This effect would cause higher temperatures to be pushed toward the tip of the fin, and, if the fin is hotter, it dissipates more heat.

To study the effects of the convecting fluid velocity on the fin array effectiveness, the velocity was changed from 5 to 15 m s^{-1}. Comparison with the 5 m s^{-1} results showed that, although the actual heat dissipated by the fin array with increased velocity increased, the fin array effectiveness decreased. Again, this is due to the way fin effectiveness was defined and caused by the fact that the heat flux from the base without fins increased as fluid velocity increased. In general, it is less important to consider emissivity effects when the convection is high because radiation will contribute to a smaller percentage of the total heat loss.

To examine the effect of fin spacing on the effectiveness of the array, the spacing was changed from 2 cm to 5 cm and the effectiveness of the fin array was plotted against β for $\varepsilon = 0$, 0.1, and 1 for $V'' = 1\times10^{-4}$ m^3 m^{-2} in Fig. 4(a) and for $V'' = 4\times10^{-3}$ m^3 m^{-2} in Fig. 4(b). Comparison of Fig. 4(a) with 3(a) and Fig. 4(b) with 3(b) shows that increasing the fin spacing (decreasing the number of fins) decreases the effectiveness of the array.

Due to the recent interest in closely packed "micro" fins, the effectiveness of fin arrays with $\delta = 0.5$ cm was also looked at. Graphs of the fin array effectiveness vs L/δ for various values of V''/δ are presented in Fig. 5(a) for $\beta = 0$ and in Fig. 5(b) for $\beta = 1.5$. Note that, in both Fig. 5(a) and (b), with the higher values of V''/δ it is harder to find a local maximum for the fin array effectiveness. The effectiveness starts high and continually decreases with increasing L. This is prob-

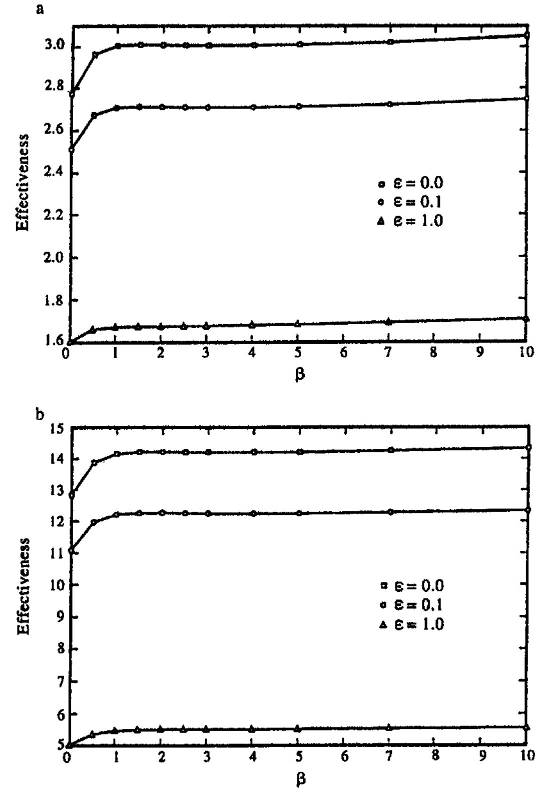

Fig. 4. Optimal fin array effectiveness versus β for air convecting over aluminum fins with $\delta = 5$ cm, $v = 5$ m s^{-1}, $T_a = 300$ K, $T_b = 500$ K. (a) $V'' = 1 \times 10^{-4}$ m^3 m^{-2} and (b) $V'' = 4 \times 10^{-3}$ m^3 m^{-2}.

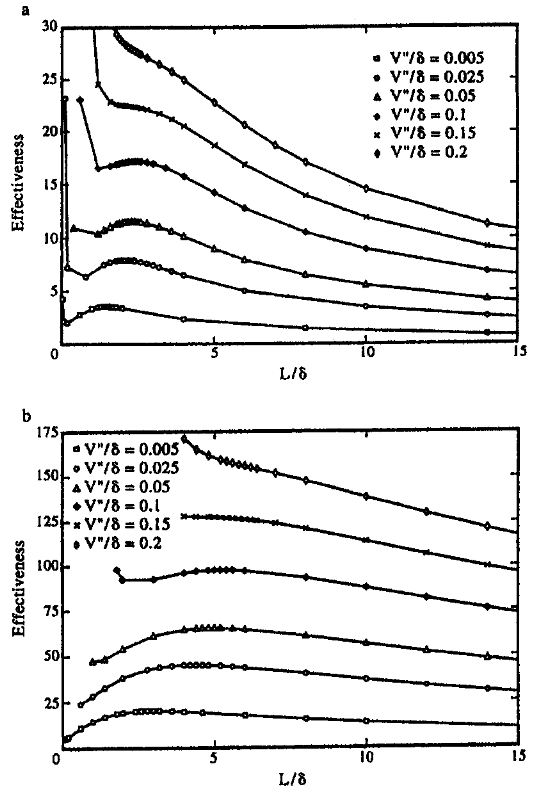

Fig. 5. Fin array effectiveness vs L/δ for air convecting over aluminum fins with $\varepsilon = 0.1$, $\delta = 0.5$ cm, $v = 5$ m s^{-1}, $T_a = 300$ K, $T_b = 500$ K. (a) $\beta = 0$ and (b) $\beta = 1.5$.

ably due to the optimum L causing the fin to be too close to the other fins, which creates a large convection coefficient. This occurrence was discussed earlier. When the optimum fin falls in this region, it gets "lost" in the noise caused by the large convection coefficients as $r \rightarrow \delta/2$. As discussed earlier, the pumping power required to create these large convection coefficients will probably make these fin arrays prohibitive.

The fin array effectiveness for the surrounding cylinder radius as calculated by the method presented in the theory section was compared to the results found by using a surrounding cylinder radius equal to the fin spacing and to a very large radius (10 m). The large value of R accounts for fin to base interactions, but fin to fin interactions are negligible. For air convecting over aluminum fins with the emissivity equal to 1 (this represents the maximum radiation effects), $V'' = 4 \times 10^{-3}$ m^3 m^{-2}, $\delta = 2$ cm, $v = 5$ m s^{-1}, $T_a = 300$ K and $T_b = 500$ K, the optimal effectiveness vs β is investigated using the three different methods of calculating R. There is a noticeable difference in the curves, but it is not very large. The surrounding cylinder radius, as calculated by the original technique, was larger than the fin spacing. As the surrounding cylinder radius was increased, fin to fin interactions became negligible and increased the effectiveness of the fin array. For other geometries, the effect of how the surrounding cylinder is estimated may have greater effects.

Calculations were also performed for stainless steel fins with $\beta = 0$ and $\beta = 1.5$; $V'' = 1 \times 10^{-4}$ m^3 m^{-2}; and $\varepsilon = 0$, 0.161, 0.254 and 1. The lower conduction coefficient of stainless steel decreases the effectiveness of the array compared to aluminum. The results are summarized in Table 2. Comparison of row 1 to row 4 and row 5 to row 8 shows that the effect of considering radiation could result in an increase in the heat flux dissipated by 27 and 29%, respectively. Comparing these results to those in Table 1 for the aluminum fins shows that the effects of radiation heat transfer are greater for the stainless steel fins (lower thermal conductivity). This phenomenon is caused by the radiation pushing hotter temperatures toward the tip of the fins. The inclusion of mutual irradiation among the fins is effectively equivalent to increasing the thermal conductivity of the material. Therefore, its effect is more important for fins with low thermal conductivity.

CONCLUSIONS

This study found that tapering the fin so that more volume was on the bottom of the fin than on the top of the fin increased the effectiveness of the fin array for a given VPUA. However, increasing the fin profile parameter beyond a value of about 1.0 did not significantly effect the fin array effectiveness. The significance of this result is that fins with high curvature, which are hard to manufacture, do not produce significant advantages to warrant their use. But a simple triangular profile fin produced significant increases in fin array effectiveness compared to a rectangular (straight) profile fin.

This study found that as the fin spacing was decreased, the effectiveness of the array increased rapidly for a fixed VPUA. In addition to the manufacturing limitations that might be imposed, this study found evidence that the large effectiveness caused by closely packed arrays may be due to large convection coefficients requiring large pumping powers. This illustrates that minimizing the VPUA is not the only factor to consider when designing a fin array.

Considering the emissivity effects of the fins and the base plate turned out to have a larger effect on the fin array effectiveness for low values of VPUA and for low values of the fin conduction coefficient. These effects may be attributed to the effect radiation heat transfer has in transferring heat to the top of the fin, pushing the tip of the fin to higher temperatures. These results may also be attributed to lower values of convection or conduction heat transfer. When the other modes of heat transfer are small, radiation effects will produce a greater effect on the ability of the array to dissipate heat. This was evidenced by the emissivity of the fins producing less of an effect for an array of fins with a higher velocity.

The effect of increased fin emissivity, with constant base emissivity, tended to increase the effectiveness of the fin array more if the emissivity of the base was small and if the fin profile parameter was large.

This study found that it was not critical to be able to estimate the radius of the equivalent surrounding cylinder more accurately. Noticeable differences did, however, show up. For cases where radiation plays a more significant role, the discrepancies would be larger. The estimate that was used to predict the radius of the surrounding cylinder was shown to fall between the two extremes of using a surrounding cylinder radius equal to the fin spacing and ignoring fin to fin interactions by considering radiation heat transfer from the fins to the base and atmosphere only.

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